Wave-Particle Duality of Electrons Lab Report*

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Abstract: Upon the first discoveries of the electron in 1897, J.J Thomson has well established the particle nature of electrons through a series of experiments on a Cathode Ray Tube (CRT) studying the electron deflection in the electric and magnetic fields. With the development of quantum mechanics in the early 20th century, de Broglie proposed that all microparticles must have wave-like properties. The wave nature of electrons was first directly observed by Davisson and Germer's electron reflection diffraction experiment in 1927. The wave-particle duality is now the embedded foundation of quantum mechanics. In this lab, we performed three experiments. Two of the experiments were similar to Thomson's CRT to study the electron trajectories in both electric and magnetic fields, which implied the particle nature of the electron. By measuring the trajectories of the electron, we estimated the e/m ratio. The other experiment indicated the wave nature of the electron that was similar to that of Davisson and Germer by using a polycrystalline gold foil target between an electron gun and a screen. The interference of electrons was directly observed on the screen. By measuring this diffraction pattern, we can estimate Planck's Constant and verify the de Broglie equation.

I. INTRODUCTION

In the 1800s, the atomic theory proposed by John Dalton was widely accepted by scientists that all matters consist of tiny indivisible particles called atoms[1]. In the late 1800s, the atomic theory was challenged by the Cathode Ray Tube Studies. The Cathode Ray tube was initially produced by the assistant of German Physicist Julius Plücker to improve the vacuum tube [2]. He found a green glow on the tube wall and discovered the cathode rays from the electrodes by placing two electrodes inside the vacuum tube. In 1879, British physicist and chemist William Crookes took a closer investigation on cathode rays and found that the rays were bent by the magnetic field. The direction of the deflection suggested that the rays were negatively charged particles. Because of his discoveries, the cathode rays tube was named Crookes' Tube and was widely studied by scientists. However, he couldn't prove whether the cathode rays were particles or electromagnetic radiations because the cathode rays were not affected by gravity. In 1892, German physicist Heinrich Hertz found that the cathode rays were unaffected by the electric field in an experiment, which suggested that the rays were radiations similar to light. In 1897, British physicist J.J Thomson repeated Hertz's experiment using a better vacuum tube with two aluminum plates parallel to the cathode rays on the top and bottom. He discovered that when the upper plate was negatively charged, the cathode rays were bent downwards; when the upper plate was positively charged, the cathode rays were bent upwards. This was a crucial discovery that complemented the discovery of deflection on magnetic fields, which made it clear that the cathode rays were negatively charged particles. J.J Thomson then called this particle electron. He calculated the charge to mass ratio of the electron and its speed by measuring the magnitude of the deflection by both magnetic and electric fields. He compared the charge to mass ratio of an electron to that of an atom which was once believed to be the smallest particle, and found that the mass of an electron is 1000 lighter than an atom. This was a groundbreaking discovery as it destroyed the long-lasting belief that atoms were the smallest particles. The discovery of the electron earned J.J Thomson the 1906 Nobel Prize in Physics "in recognition of the great merits of his theoretical and experimental investigations on the conduction of electricity by gases." [3]

In the early 20th century, Einstein's discoveries of the photoelectric effect and the development of Quantum Mechanics unraveled the particle nature of light that was once believed to be a wave phenomenon. Inspired by the duality of light, as a young graduate student at Paris University in 1924, Louis de Broglie delivered a thesis on quantum theory containing his novelty thought that electrons and other "particles" might exhibit wave properties[4]. This concept was known as the de Broglie hypothesis. Early in 1924, American physicists Clinton Davisson and Lester Germer were studying the surface of a piece of nickel by directing a beam of electrons at the surface and observing how many electrons bounced off at various angles[5]. In 1926, Davisson attended the Oxford meeting of the British Association for the Advancement of Science and learned of the recent advances in quantum mechanics. In 1927, with a better knowledge of de Broglie's formula, Clinton Davisson and Lester Germer fired electrons with different energy from the electron gun to a crystalline nickel target. They found the angular dependence of the maximum intensity of electrons diffracted by the atomic surface. From Bragg's condition for constructive interference from an array, they calculated the corresponding wavelength for the observed pattern from the electrons with a kinetic energy of 54 eV via Bragg's law. The experimental outcome was 0.165 nm, which closely matched the predictions of 0.167 nm by the de Broglie relation[6]. The Davisson-Germer experiment confirmed the de Broglie hypothesis that electrons have wave-like behavior. At the same time, British

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Physicist George Paget Thomson independently demonstrated the same effect by firing electrons through metal films to produce a diffraction pattern. In 1929, de Broglie won the Nobel Prize in Physics "for his discovery of the wave nature of electrons." Clinton Joseph Davisson and George Paget Thomson were awarded The 1937 Nobel Prize jointly in Physics "for their experimental discovery of the diffraction of electrons by crystals."[7]

In this Lab, we reproduced J.J Thomson's experiment on CRT, studying the particle nature of electrons, and Davisson-Germer's experiment on electrons diffraction, exploring the wave nature of electrons. We will introduce the theory that our experiments are dependent on in section II. Then we will demonstrate our experiments' procedure and present the data analysis in sections III and IV.

II. THEORETICAL CONSIDERATIONS

A. Electron Charge-to-Mass Ratio: Particle Property

According to the Lorentz force law, a particle with charge q moving with a velocity v in an electric field E and a magnetic field B experiences a force of

$$F = qv \times B + qE \tag{1}$$

The electron is a subatomic particle whose charge is negative one elementary charge $e = 1.602 \cdot 10^{-19}C$. Thus, the electric force on the electron moving in electric field v is:

$$F_e = -eE \tag{2}$$

The electron can be accelerated through a uniform magnetic field with a voltage difference of V in the opposite direction of the field. The energy gain for the electron is:

$$Energy = eV \tag{3}$$

According to the conservation of energy:

$$\frac{1}{2}mv^2 = qV \tag{4}$$

, where m is the mass of the electron. Thus the velocity of the electron after the accelerating is:

$$v = \sqrt{\frac{2qV}{m}} \tag{5}$$

If we send this electron into a uniform magnetic field, it will experience a Lorentz force of:

$$F_m = -ev \times B \tag{6}$$

The direction of the force is always perpendicular to the trajectory of the electron. Thus, the electron will move along a circular path in the uniform magnetic field. Figure 1 shows the circular path for an electron. According to Newton's Second law on the circular motion, the centripetal force is

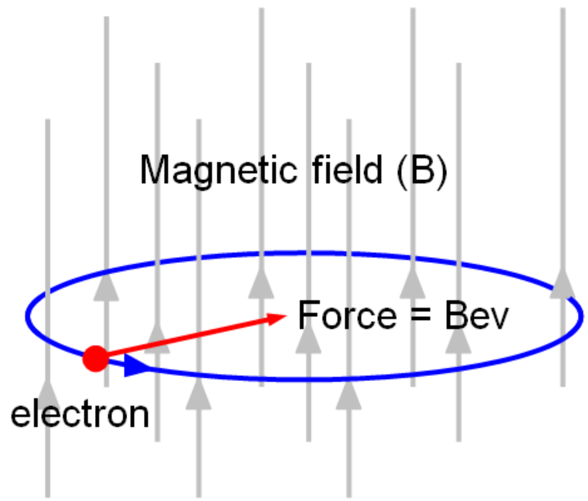

FIG. 1. The circular path of an electron moving in a uniform magnetic field. Image Credit: [8]

$$F_c = m \cdot \frac{v^2}{r} \tag{7}$$

, where r is the radius of the circular path. The magnetic force is equal to the centripetal force. Therefore, the charge to mass ratio can be expressed as:

$$\frac{e}{m} = \frac{2V}{B^2r^2} \tag{8}$$

1. *The Cathode Ray Tube*

J.J Thomson used a Cathode Ray Tube to study the particle properties of electrons. Figure 2 shows the Cathode Ray Tube used by Thomson. The electrons were accelerated by the High voltage on the left. The electrons then traveled through adjustable magnetic and electric fields and hit the screen on the left. The path of the electrons can be deflected both my the magnetic and electric fields. Figure 3 shows the deflection of an electron by the external uniform electric field. The electron in the electric field experiences a Lorenz force $F_y = eE_y$ and spends the time of $\delta t = w/v_e$ in the field. The change of momentum is

$$mv_y = F_y\delta t = eE_y w/v_e \tag{9}$$

,where w is the width of the electric field with a strength of $E_y = V_d/d$. Assume that the electron was accelerated by a accelerating voltage V_k. The energy of the electron is then

$$1/2mv^2 = eV_k \tag{10}$$

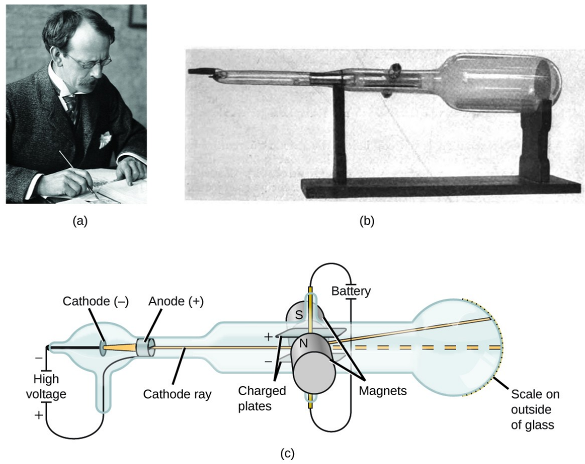

FIG. 2. The Cathode Ray Tube used by J.J Thomson. Image Credit: [9]

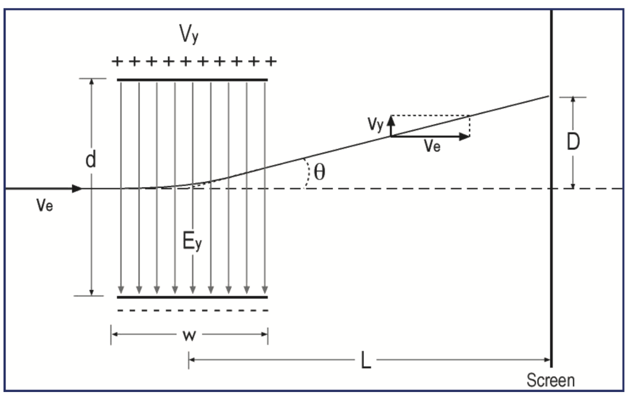

FIG. 3. The deflection of electron with velocity v_e by the electric field E_y. D should be proportional to the strength of the field. Image Credit: [10]

Substituting v_y and v_e in $tan\theta = v_y/v_e = D/L$, we have

$$D = \frac{wL}{2dV_k} \cdot V_y \tag{11}$$

Thus, the distance of the dot image on the screen is further from the middle point if the charged plates have a higher potential difference when the magnetic field is zero. Similarly, if the electric field is zero, the degree of deflection should also proportional to the strength of the magnetic field. Figure 4 shows the deflection of an electron by the external uniform transverse magnetic field. The Lorentz force is perpendicular to the velocity with a magnitude of evB. The trajectory of the electron in the magnetic field is a segment of a circle with radius R with centripetal force of

$$mv^2/R = evB \tag{12}$$

The electron path is deflected through an angle ϕ by the magnetic field, and $sin\phi = b/R$. After the electron leaves

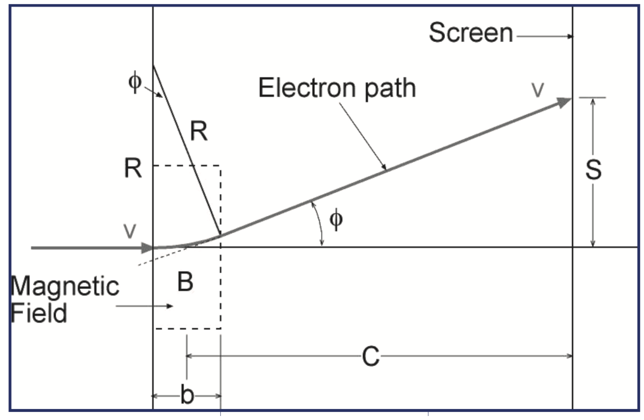

FIG. 4. The deflection of electron with velocity v_e by the magnetic field B in the direction of pointing out of the page. D should be proportional to the strength of the field. Image Credit: [10]

the magnetic field, it follows a straight line. Projecting the straight path backward, it meets the midpoint of b with an angle ϕ. Thus, $tan\phi = S/C$ and we have

$$S = Ctan\phi = C\frac{sin\phi}{\sqrt{1 - sin^2\phi}} \tag{13}$$

The electron was also accelerated by voltage V_k, using equation 10 for v and equation 12 for R, we obtain

$$S = Cb\sqrt{\frac{e}{2m}}\frac{B}{\sqrt{V_k}} \tag{14}$$

The magnetic field is provided by the current i through the coils: $B = Ki$, where K is a constant. So the equation 14 becomes:

$$S = KCb\sqrt{\frac{e}{2m}}\frac{i}{\sqrt{V_k}} \tag{15}$$

Thus, the deflection S is proportional to the current i and inversely proportional to $\sqrt{V_k}$ If we apply longitudinal magnetic field throughout the tube, the electron will travel in a spiral path shown in figure 5. Let v_R be the radial velocity of the electron and v_z be the velocity in the direction of magnetic field. The distance p between each pitch is $p = v_z T$, where T is the period of the path given by $T = 2\pi R/v_R$. Substituting in T and use the equation 10 with $v = v_z$, we obtain:

$$p = \sqrt{\frac{2eV_k}{m}}2\pi\frac{m}{eB} \tag{16}$$

When the spot is focused on the screen, p = L/n (n = 1, 2, 3 for the first, second, and third time a focus point is found when increasing the magnetic field). Rearranging the equation 16 we have:

$$B^2 = \frac{m}{e}\left(\frac{8\pi^2 V_k}{L^2}\right) \tag{17}$$

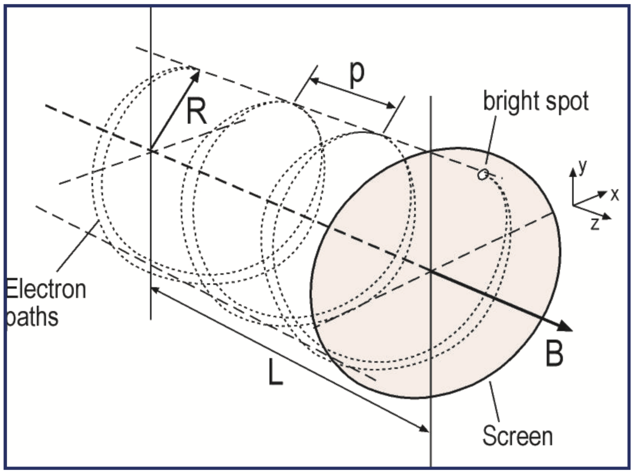

FIG. 5. The spiral path of electron with velocity v_e by the magnetic field B in longitudinal direction. Image Credit: [10]

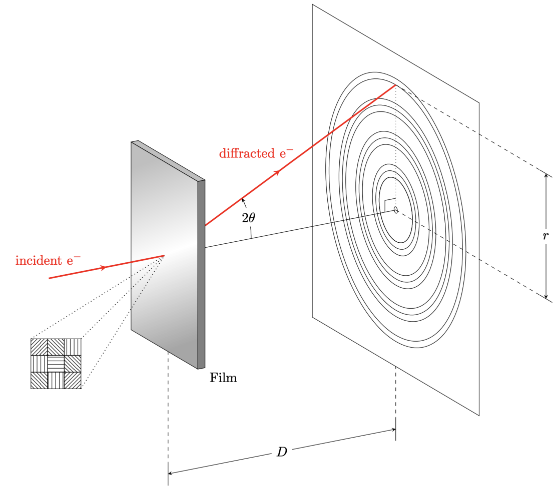

FIG. 6. The deflection pattern image on the screen Credit: [11]

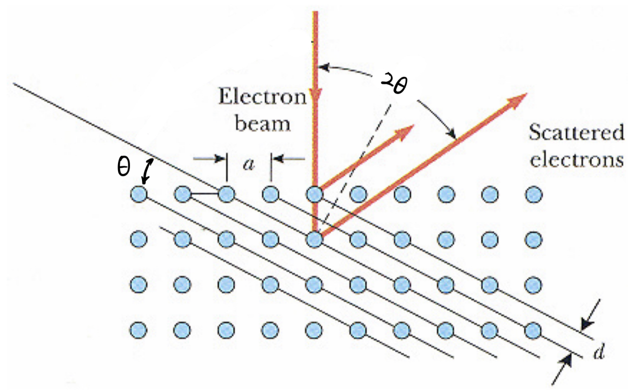

FIG. 7. The electron beam deflected by the crystal target in an angle 2θ. d is the Brags length for the crystal and it can be calculated if we know its atomic structure. a is the lattice constant. Credit: [11]

We can obtain the strength of the magnetic field by

$$B = 4\pi Ni \times 10^{-7}/\sqrt{D^2 + L_s^2} \tag{18}$$

, where N, D and L_s are the number of the turns, the diameter, and length of the solenoid carrying a current i. Then we can use the equation 17 to determine the electron-mass ratio for electrons.

B. Electron Diffraction: Wave Property

de Broglie combined the Einstein's mass energy function $E = mc^2$ and Planck's theory $E = h\nu = hc/\lambda$ to derive the wavelength equation for a particle with speed v substituting the speed of light c:

$$\lambda = \frac{h}{mv} = \frac{h}{p} \tag{19}$$

, where p is the momentum of the particle. This equation is known as the de Broglie wavelength. His theory was confirmed by the Davisson-Germer experiment using the electron diffraction tube. Inside the tube, a beam of electrons emitted by the electron gun hit on Nickel crystal and deflected by the target in an angle of $2*\theta$ determined by the structure of the crystal(See figure 5). The path difference between the adjacent diffraction rays needs to be exactly equal to the incident wavelength so that they can have constructive interference and make a maximum light pattern on the screen, confirming the wave nature of the moving electrons (see Figure 7).

The constructive interference requires:

$$n\lambda = d\sin\theta \tag{20}$$

The Miller indices can be used to describe certain direction and planes. Figure 8 shows the Crystallographic directions and planes. The electron can be deflected in different direction based on this crystal structure. Figure 9 gives some example of how Miller indices can describe certain direction and planes. Knowing the Miller indices (h, k, l) and the lattice constant s, the Brads length d is:

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \tag{21}$$

Substituting the Brags length and $sin\theta = r/D$, equation 20, and let $H = nh$, $K = nk$, $L = nl$, the equation 20 becomes:

$$\lambda = \frac{r}{D}\frac{a}{\sqrt{H^2 + K^2 + L^2}} \tag{22}$$

Comparing this experimental results with the de Broglie wavelength equation 19 for electron with velocity $v = \sqrt{2eV/m}$ (see equation 5), we can estimate the Planck

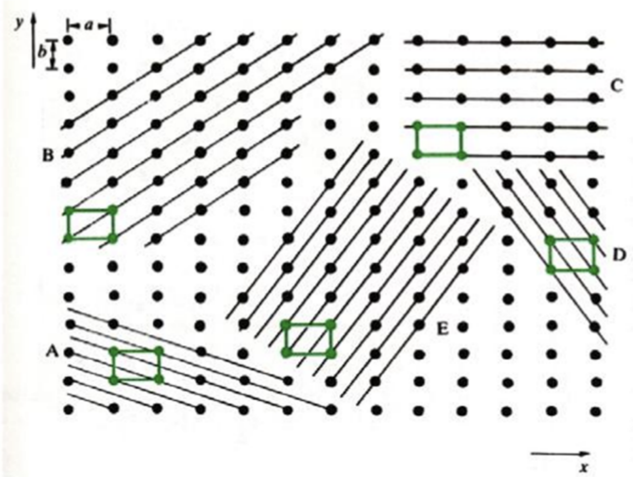

FIG. 8. The Crystallographic directions and planes. This explains the different possible angles of deflections. Credit: [12]

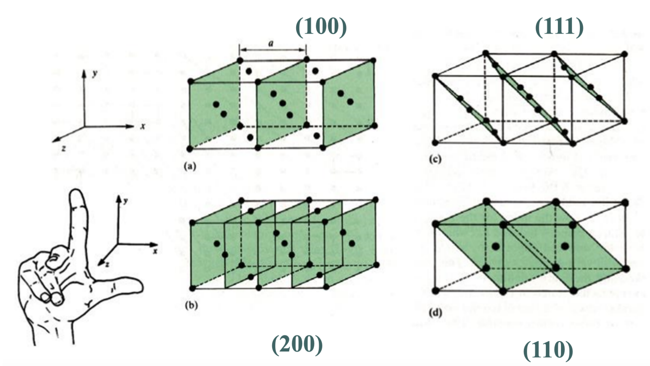

FIG. 9. Examples of some Muller Indies. Credit: [12]

constant:

$$h = \lambda * \sqrt{2meV} \tag{23}$$

III. EXPERIMENTAL METHODS

A. The Electron Charge-to-Mass Tube

The first experiment we performed is measuring the electron Charge-to-Mass ratio using the equipment showed in the figure 10. The electron beam is accelerated by the external DC Power supply and is immersed into the longitudinal magnetic field created by the Helmholtz Coils. We set the accelerating voltage from 90 to 300V with step of 30V. For each voltage value, we slowly increase the current to increase the magnetic field. The videos were token for each different voltage to make the measurement of the radius. We directly measured the strength of the magnetic field using the magnetic probe. Figure 11 showed a screen shot of how we measured the

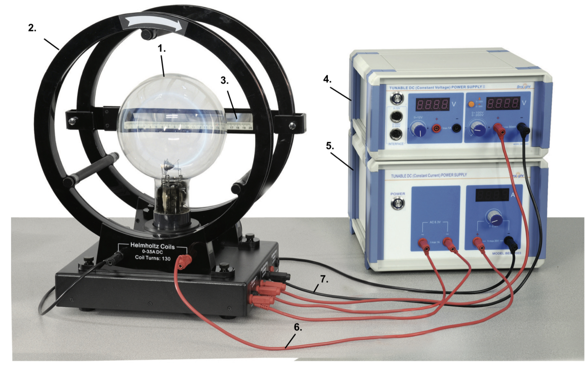

FIG. 10. Equipment for the first experiment. 1.e/m tube; 2.Helmholtz Coils

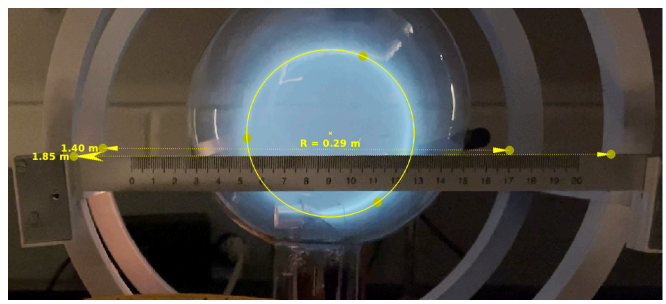

FIG. 11. Example measurement for V=150V, B = 0.8mT. The unit of m means the length on in the picture, not the actual length

radius of the electron beans using the PASCO Capstone video measurement tools. Because of the perspective, the actual radius was distorted by a particular factor. We use a ruler to measure the actually diameter of the circular base $d = 26.6cm$. In the PASCO Capstone, we measure the two circular length in the picture 11 D_1, D_2, and the radius of the electron beam R. The diameter of the circular base have the different measured length because of the line of sight. Thus, its easy to derive that the actual radius of the electron beam should be:

$$r = R * 1/2(\frac{d}{D_1} + \frac{d}{D_2}) \tag{24}$$

We have recorded 8 videos with different accelerating voltage $V = 90, 120, 150, 180, 210, 240, 270, 300V$ with increasing magnetic field. Each video, we measured the radius of the radius for as much as 9 different value of magnetic field $B = 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8mT$. We will showed our measurement results and their analysis in the next section.

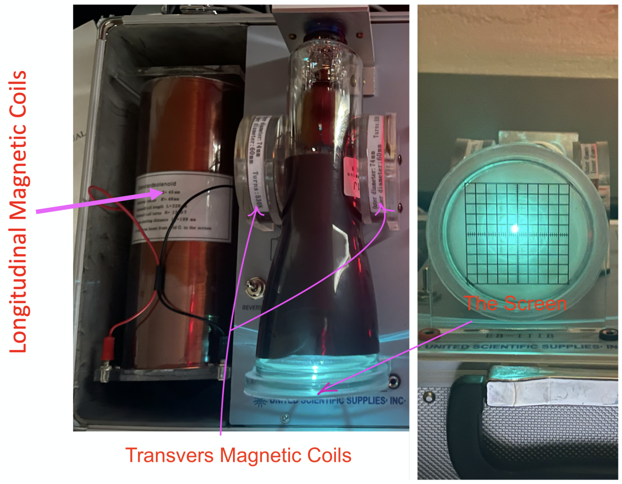

FIG. 12. The Cathode Ray Tube

B. The Cathode Ray Tube

In this experiment, we performed a thorough study on the particle nature of electrons using a Cathode Ray Tube that can be immersed into transverse and longitudinal magnetic fields. Figure 12 shows the materials we used in this experiment. The electron was accelerated by the voltage V_k and can be deflected by the electric field V_y and V_x in vertical and horizontal directions. The focus voltage V_I can be adjusted to ensure a sharp dot image on the screen. The two magnetic coils can be connected to the adjustable current i respectively to create a transverse magnetic field or longitudinal magnetic field.

1. First Run

For the first run, we accelerated the electron by applying $V_k = 1000, 1200, 1350V$ respectively and gradually increased the V_y. We recorded the different values of V_y corresponds to the the ten different vertical positions of the light spot on the screen $D = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$ for all the three different accelerating voltages.

2. Second Run

For the second run, we applied two additional anodes to create an inhomogeneous electric field, known as the electron lens. Figure 13 shows the paths of the electrons through the inhomogeneous electric field created by the two anodes. By adjusting the focus voltages V_I on FA/A_2 and $A1$, the position of the convergence point F_2 can be changed. Another anti voltage V_G was applied right after the electron got accelerated. We recorded the different values of V_G and V_I for a sharp spot on the screen with three different $V_k = 100, 1200, 1350V$

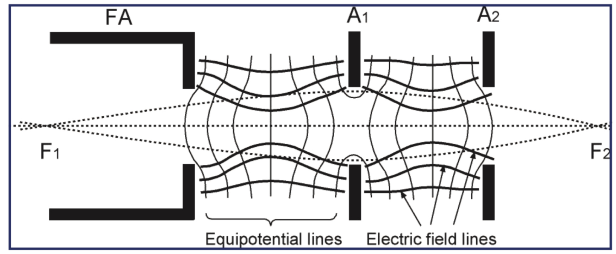

FIG. 13. the electron paths through the electron lens.

3. Third Run

For the third run, we applied the transverse magnetic field by plugging in the transverse coil shown in figure 12. The electron was deflected vertically. We recorded the different values of the current that was connected to the coil corresponding to ten different vertical positions of the bright spot on the screen for three different accelerating voltages $V_k = 1000, 1100, 1200, 1350$.

4. Fourth Run

For the fourth run, we applied both the longitudinal magnetic field and the electric field to estimate the charge to mass ratio using the theory in equation 17. We recorded the current that was connected to the coil corresponds to the first appearance of the bright spot on the screen for three different accelerating voltages $V_k = 1000, 1100, 1200, 1300V$. The instrument constants are $L = 0.199m$, $N = 1300$, $D = 0.0945m$, and $L_s = 0.235m$.

C. Electron Diffraction

We use the electron diffraction tube similar to the tube used by Davisson and Germer with gold foil (lattice constant $a = 0.40786nm$) as a target. The distance between the target and the screen is $D = 255mm$. We applied 10 different accelerating voltage $V = 11, 12, 13, 14, 15, 16, 17, 18, 19, 20V$ and measured the radius of the diffraction patterns for different Miller indices shown in figure 14. For each accelerating voltage, we obtained the average wavelength for these eight different measurements and estimated the Planck constant using the equation 23

IV. DATA ANALYSIS

A. The Electron Charge-to-Mass Tube

The measurement results for the radius of the electron beams are shown in figure 15. From this figure, we can see

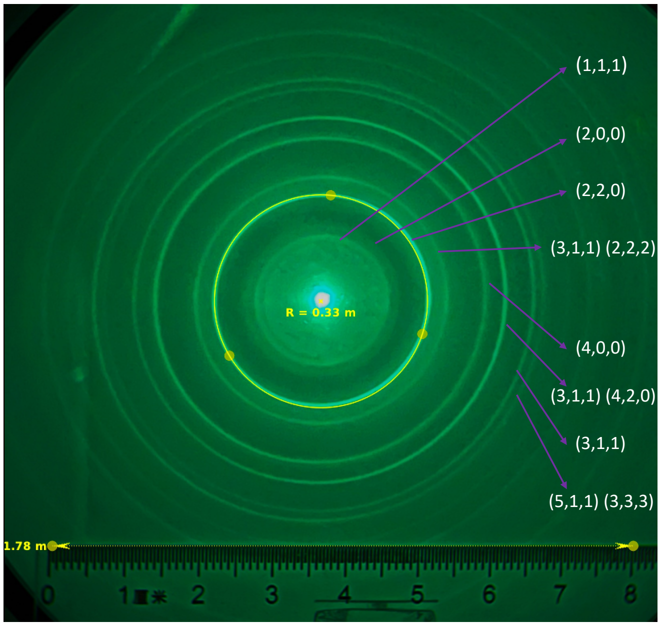

FIG. 14. Measurement method example for electron diffraction with V = 20V. The actual radius is $r = R/1.78(mm)$ in this particular case

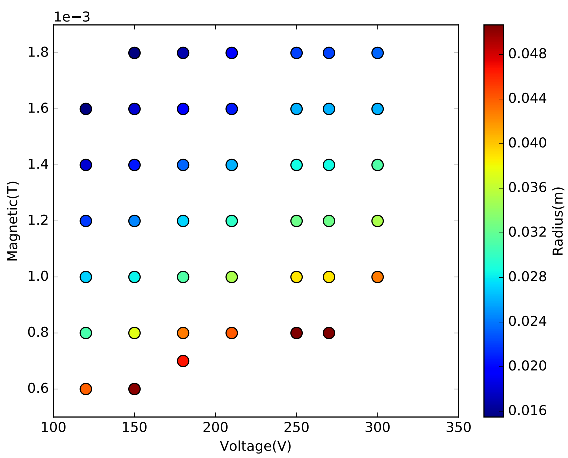

FIG. 15. The measurement results for radius of the Electron Charge-to-Mass Tube

that the radius is proportional to the accelerating Voltage and inverse proportional to the strength of Magnetic field, in agreement with the theory equation 7. We plotted the The $1/B^2$ vs $r^2/2V$ graph and fit a straight line for the data. We obtained a e/m value of $1.97 \cdot 10^{11} C/kg$ with an error of 12% (theoretical value: $1.76 \cdot 10^{11} C/kg$).

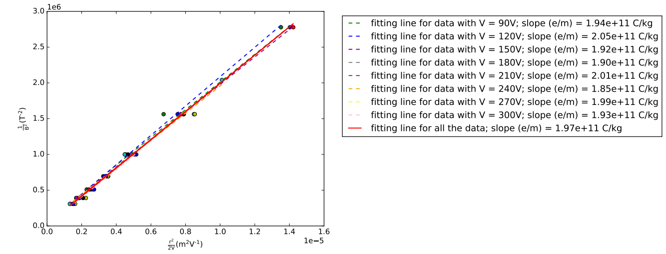

FIG. 16. The slop is the e/m value according to equation 7

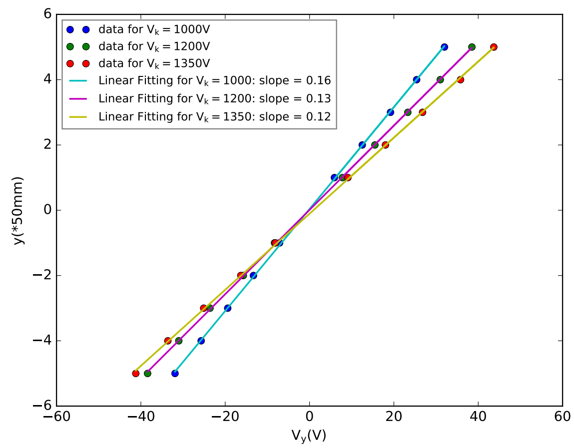

FIG. 17. Data and fitting line of D vs. V_y

B. The Cathode Ray Tube

1. First run

We plotted the graphs of D vs. V_y recorded in the first run. Figure 17 shows that the strength of electron deflection is proportional to the strength of the external electric field and is inverse proportional to the velocity of the electron. That is, the faster the electron, the less the defection of the electron would be. These are in agreement with the theoretical equation 11.

2. Second run

In figure 18, we can see that the focus voltage V_I needs to be stronger for higher accelerating voltage V_k. This indicates that the faster the electron, the less the defection of the electron would be. Thus, it needs a higher focus voltage to converge the electron beam. For the same accelerating voltage V_k, the focus voltage V_I has an inverse relationship with V_G. This is not intuitively correct since the higher negative of V_G, the slower the electron would be, and hence the lower focus voltage V_I should be applied. This may be due to the limit of human eyes when the V_G has lower negative values; it's tough

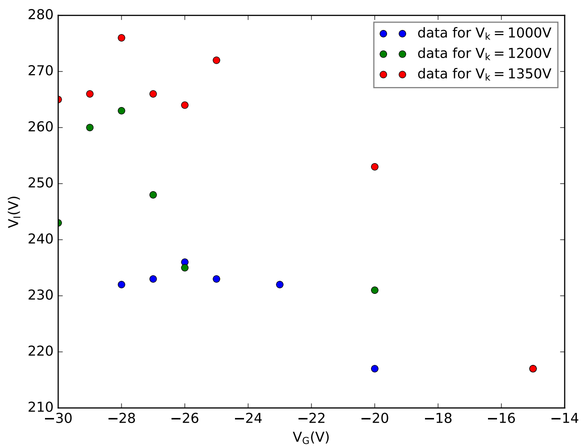

FIG. 18. Data of V_I vs. V_G

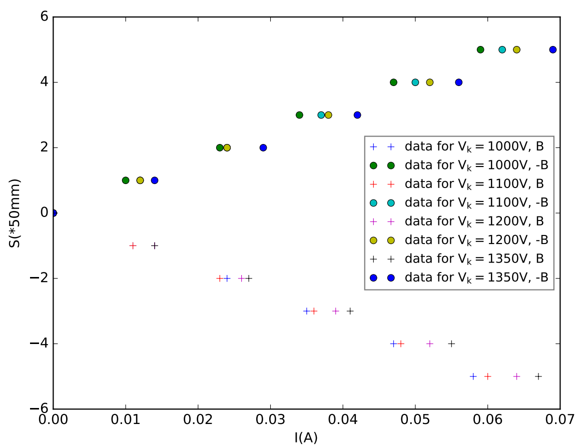

FIG. 19. Data of S vs. I

to tell whether the image is sharper or not because the image is dim. Thus, the data points for the lower negative V_G have large errors. If we exclude these points for $V_G > -20$, we can't tell the trend of the V_I related to V_G. This may be due to the change of V_G being too small to make a detectable change on V_I.

3. Third run

In figure 19, we plotted the data of position of the image on the screen S and the strength of the current that was applied to the transverse magnetic coil. We can tell from the figure that S is proportional to the current and inverse proportional to the V_k, in agreement of the theory equation 14.

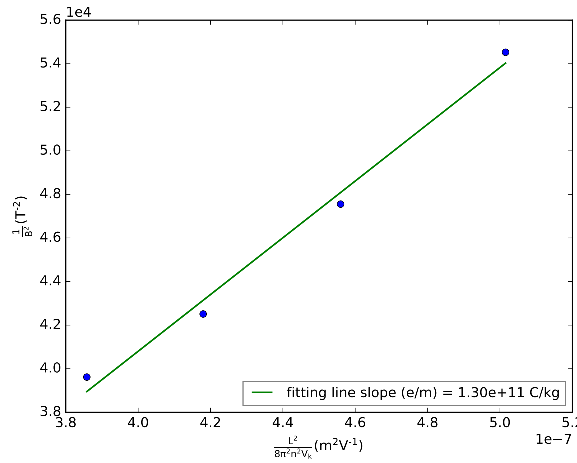

FIG. 20. Data and the fitting line for $\frac{1}{B^2}$ vs. $\frac{L^2}{8\pi^2 V_k}$. Magnetic field strength B was calculated by equation 18 using the instrument constant showed in section III B 4. The slope of the fitting line is the e/m according to the equation 17

4. Forth run

In this run, we recorded the strength of the current that was applied to the longitudinal magnetic coils corresponding to the first time a focus point appeared on the screen for all the four different values of the accelerating voltage. Then we use the equation 18 to calculate the strength of the magnetic field with recorded current i. We plotted the data and fitting line for $\frac{1}{B^2}$ vs. $\frac{L^2}{8\pi^2 V_k}$ in figure 18. The slope of the fitting line is the e/m according to the equation 17. The estimation $(1.3 \cdot 10^{11} C/kg)$ is close to the theoretical value $(1.76 \cdot 10^{11} C/kg)$ with an error of 26%.

C. Electron Diffraction

The measurement data of the radius and calculation results of the average electron wavelength for accelerating voltage $=$ 20V are shown in the table I. We used the same measurement and calculation method for all the other ten different values of accelerating voltage $(V = 11, 12, 13, 14, 15, 16, 17, 18, 19, 20)$. We plotted the average values of the electron wavelength vs. the voltage in figure 21. The slope of the best fitting line is $\frac{h^2}{2me}$ according to the equation 23. Thus we estimated the Planck constant $h = 6.00 \cdot 10^{-34}$ with an error of 9.4%. (theoretical Planck constant $h = 6.63 \cdot 10^{-34}$)

V. CONCLUSION

Through these three experiments, the particle and wave nature of electrons was well explored. All the experiment results agreed well with the theory. One thing

TABLE I. Measurements and calculations of wavelength of the electrons for accelerating voltage = 20V

(h, k, l)	r(mm)	λ(pm)
1 1 1	8.99	8.30
2 0 0	10.34	8.27
2 2 0	14.83	8.39
3 1 1	17.08	8.24
4 0 0	22.47	8.99
3 3 1	25.17	9.24
4 2 2	29.21	9.54
5 1 1	30.56	9.41
average	N/A	8.80

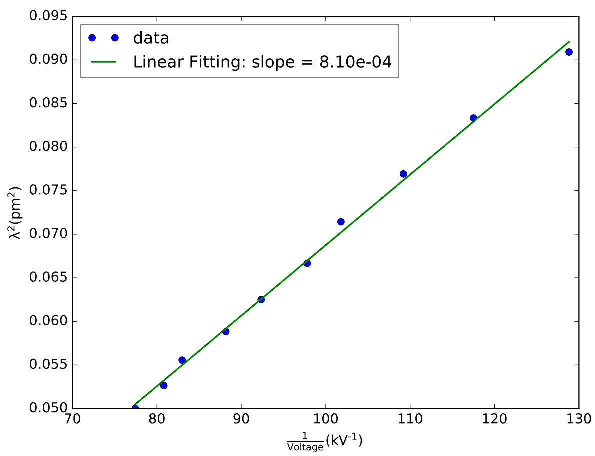

FIG. 21. The average electron wavelength vs. voltage graphs. The slop is $\frac{h^2}{2me}$ according to the equation 23

needed to be noticed is that we need to consider the line of sight perspective of the picture for measurement of the radius for the electron light bulb of the first experiment. A factor should multiply the measurement radius to get the actual radius. Also, the estimation of e/m for the second experiment has a relatively large error (26%). We think this may be due to the challenge of telling whether the spot on the screen is sharp enough. We realized that we could have used the AC/DC switch on the apparatus for better measurement that would produce a line on the screen instead of a spot to better assist with discerning the sharpness.

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