Zeeman Effect Experiment Lab Report*

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Abstract: The Zeeman Effect was first experimentally confirmed by the Dutch physicist Pieter Zeeman in 1896. It is the effecting of spitting of spectral lines of atoms due to the presence of an external magnetic field. In this experiment, we studied the anomalous Zeeman Effect by transversal and longitudinal observation from a Fabry-Perot interferometer for the 546.1 nanometers (nm) spectral line of a mercury lamp that was immersed in a uniform adjustable magnetic field. A polarizer and a quarter-wavelength plate were used to select the polarization state of the light. A triplet was observed by adjusting the polarizer parallel to the magnetic field, and double triplets (doublet) were observed by adjusting the polarizer perpendicular to the magnetic field when light emitted perpendicular to the magnetic field. Clockwise and anticlockwise circularly polarized spectral were observed by using the quarter-wavelength plate in the direction of the magnetic field. By using Lorentz's theoretical argument for electrons, we can predict the state of polarization of the light and get the accurate estimation of the Bohr Magneton, through the quantitative measurement of the observed spectral.

I. INTRODUCTION

In the summer of 1896 in Leyden University, while he was studying the influence of magnetism on the nature of light radiation [1], the Dutch physicist Pieter Zeeman noticed that the spectrum line from a sodium flame got broadened when it was placed between the poles of a powerful electromagnet[2]. Further analysis for the broadened lines discovered by Zeeman showed that it was the splitting of the spectral line into more than ten components[3], which was known as the anomalous Zeeman Effect. Zeeman's experiment directly confirmed the argument of Lorentz's classical theory of the electron, where the spectral splitting was explained by the oscillation of electron - the elementary particle that was soon confirmed by J. J. Thompson in 1897. This breakthrough earned Zeeman and Lorentz the 1902 Nobel Prize in physics. In the 1900s, when the electron spin was introduced, the normal Zeeman effect, which requires the total spin of the electron to be zero, was realized to be actually the exception for the anomalous Zeeman Effect.

The Zeeman effect has helped physicists verify the energy levels of atoms and classify their mass and spin. It is also a useful method for evaluating atomic particle and electron paramagnetic resonance and is essential in nuclear magnetic resonance spectroscopy. The Zeeman effect is also very important to the development of quantum mechanics a hundred years later. It provides direct evidence that the orbital angular momentum (the magnetic moment of the atom) is quantized. The number of spectral lines that are splitting into from a single line helps determine the total angular momentum of the energy levels of the transition that produces the spectral line.[4]

In this experiment, we studied the anomalous Zeeman Effect by using A Fabry-Perot interferometer for the 546.1 nm spectral line of a mercury lamp in a uniform magnetic field. We will introduce the physics of the Zeeman Effect and the spectroscopy of the Fabry-Perot interferometer in section II. Then we will demonstrate the experiment procedure and present the data analysis in section III and IV.

II. THEORETICAL CONSIDERATIONS

A. The anomalous Zeeman Effect

In Lorentz's theory of electrons, the orbit of the electrons got changed due to the Lorentz force when it is moving in a magnetic field, hence the change in energy. The orientation of the magnetic field changes the electron energy in different ways. If the magnetic field is perpendicular to the electron's orbital plane (the transversal Zeeman Effect), the change of the energy can be either negative or positive depending on the direction of the motion of electrons. If the magnetic field is paralleled to the electron's orbital plane (the longitudinal Zeeman Effect), the net Lorenz force is zero, hence no change in energy. [5]. The two different orientation of Zeeman Effect is illustrated in FIG 1. The theory predicts that when a magnetic field is applied, a spectral line should split into three lines - one with positive energy change, one with negative energy change, and the one with no energy change.

The anomalous Zeeman Effect is the more general case when the total spin of the electron is not zero, hence the energy of the atomic states depends on both the electron spin and the magnetic moments of electron orbit in the external magnetic field. The transition we will use to demonstrate the anomalous Zeeman Effect is $^3S_1(6s7s) \rightarrow^3 P_2(6s6p)$ with 546.1 nm of electrons in mercury (Hg) atoms. A scheme of the energy levels of Hg is shown in FIG2. The upper "3" in the notation $^3S_1$ is the multiplicity (multiplicity $= 2s+1$, here is the triplet states) with spin $s = 1$. The subscript "1" is the quantum number j, the total angular momentum (j is a integer ranging from $l-s$ to $l+s$, where l is the quantum number for angular momentum of the orbit). The "S", "P", "D" etc... are the values for quantum number l."S"

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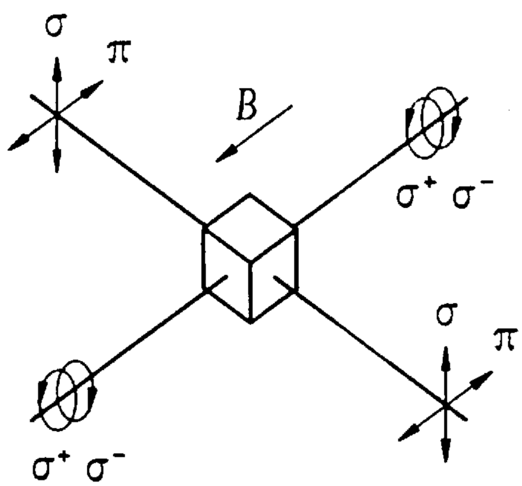


FIG. 1. The Transversal and Longitudinal Zeeman Effect

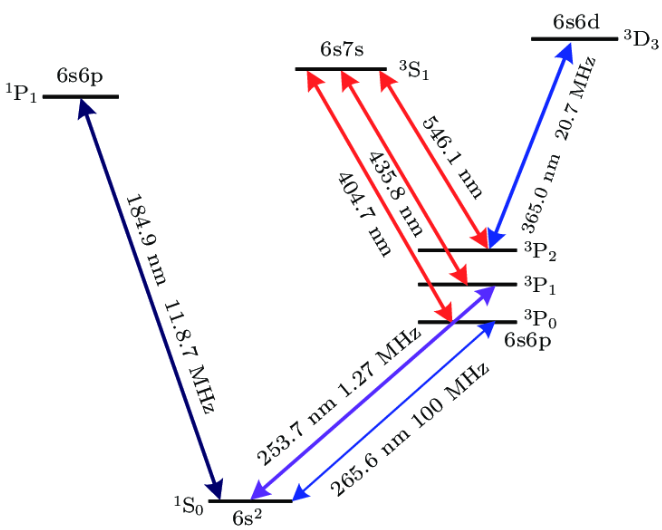


FIG. 2. Energy Levels of Hg

means $l = 0$, "P" means $l = 1$ and so on. The terms (6s7s) means there are two valence electrons one in 6s state and one in 7s state. In the presence of external magnetic field, each level, $^3S_1$ (with energy E_S) and $^3P_2$ (with energy E_P), splits into the number of $2j+1$ closely spaced levels, hence three groups of three spectral lines appear. The splitting pattern is demonstrated in FIG3[4] The selection rule for optical transitions is $\Delta M_Z = 0, \pm 1$. The transitions with $\Delta M_Z = 0$ are called π-lines and the transitions with $\Delta M_Z = \pm 1$ are called σ-lines. π-lines are polarized parallel to the magnetic field and σ-lines are polarized perpendicular to the field. In the experiment, we are using the polarizer to select the π-lines for measurement because it has the highest intensity.

The magnetic dipole moment of electrons system with the total angular momentum ($\vec{J} = \vec{L} + \vec{S}$) and the total

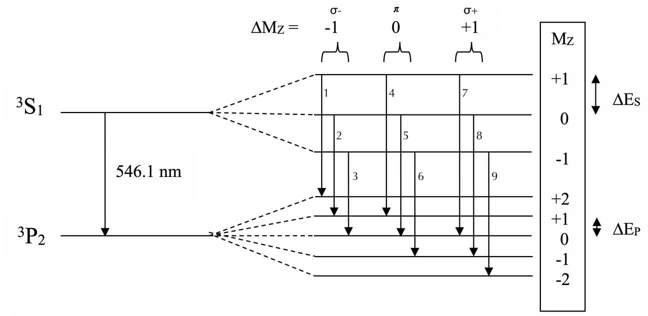


FIG. 3. Energy Levels for 546.1 nm Mercury Spectral Line in presence of magnetic field. This is not scaled. The splitting in the same group is much less than the energy difference between $^3S_1$ and $^3P_2$. M_Z is the projection of the orbital angular momentum on the z axis.

spin ($\vec{S}$) is [6]

$$\vec{\mu_J} = -g_J \frac{e}{2m_e}\vec{J} = -g_J \frac{\mu_B}{\hbar}\vec{J} = -g_J \frac{\mu_B}{\hbar}(\vec{L}+\vec{S}) = \vec{\mu_L}+\vec{\mu_S} \quad (1)$$

Hence in z-axis

$$(\vec{\mu_J})_z = -M_Z \cdot g_J \cdot \mu_B \quad (2)$$

The energy changes due to the external magnetic field B is then

$$\triangle E_{s/p} = M_Z \cdot g_J \cdot \mu_B \cdot B \quad (3)$$

, where M_Z is the magnetic quantum number with values of $J, J-1, ..., -J$, μ_B is the Bohr's magneton, g_J is the dimensionless correction factor for the classical result, known as *Landé* g-factor, $\vec{\mu_L}$ is the magnetic moment of the orbital angular momentum, and $\vec{\mu_S}$ is the magnetic moment of the spin.

$$\mu_B = \frac{e\hbar}{2m_e} \quad (4)$$

$$\vec{\mu_L} = -g_L \frac{e}{2m_e}\vec{L} \quad (5)$$

$$\vec{\mu_S} = -g_S \frac{e}{2m_e}\vec{S} \quad (6)$$

The magnitude of angular momentum is given by [7]

$$\left|\vec{L}\right| = \hbar\sqrt{l(l+1)} \quad (7)$$

The magnitude of the intrinsic spin angular momentum is also given by [7]

$$\left|\vec{S}\right| = \hbar\sqrt{s(s+1)} \quad (8)$$

Using the cosine rules, we have

$$|\vec{\mu_J}| = |\vec{\mu_L}|\, cos(\vec{L}+\vec{J}) + |\vec{\mu_S}|\, cos(\vec{S}+\vec{J}) = -g_J \frac{e}{2m_e}\vec{J} \quad (9)$$

TABLE I. Energy Shift of Each Transition Line, the energy of each spectral line is $E_0 + \Delta E$

Transition Line	1	2	3	4	5	6	7	8	9
$\Delta E/(\mu_B \cdot B)$	-1	$-\frac{3}{2}$	-2	$\frac{1}{2}$	0	$-\frac{1}{2}$	2	$\frac{3}{2}$	1

Solving the equation, we have

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \tag{10}$$

We know that S = 1, L = 0, J = S + L = 1 for state $^3S_1$ and S = 1, L = 1, J = S + L = 2 for state $^3P_2$, hence

$$g_{J(^3S_1)} = 2 \tag{11}$$

and

$$g_{J(^3P_2)} = \frac{3}{2} \tag{12}$$

According to the equation 3, the total energy shift due to the external magnetic field for a transition line between the initial states with $M_z = M_{zi}$, g_{Ji} and final states with $M_z = M_{zf}$, g_{Jf} is

$$\triangle E = E_f - E_i = (M_{zf} \cdot g_{Jf} - M_{zi} \cdot g_{Ji})\mu_B \cdot B \tag{13}$$

Let the initial energy difference be $E_0 = E_S - E_P$, hence the energy for each transition line in FIG3 after applying the magnetic field is $E_0 + \Delta E$. The magnitudes for energy shifts of each spectral line are shown in table I From the tableI, we can observe that in the three groups of triplets (123, 456, 789), the energy difference of transition lines between 1 and 3, 4 and 6, 7 and 9 are $1 * \mu_B \cdot B$. Let's use the subscript "$_-$" for the spectral lines with lower energy and "$_+$" for the ones with higher energy, and no subscript for the middle spectral line of each triplet. We can write the following:

$$h(\nu_+ - \nu_-) = \mu_B \cdot B \tag{14}$$

Converting to wavelength:

$$\lambda_- - \lambda_+ = \triangle\lambda = \frac{B\mu_B\lambda^2}{hc} \tag{15}$$

with the approximation of $\lambda_+\lambda_- = \lambda^2$

B. The Fabry-Perot Interferometer

To measure the small wavelength difference, we need a powerful tool Fabry-Perot Interferometer, which consists of two elatons (partially reflective optically flat plate of glass). The elaton has a resolution of approximately 400000 and is capable of detecting a wavelength change of less than 0.002 nm.[5]. The two elatons are separated by distance d = 1.995 millimeters (mm) in Fig4[4]. An

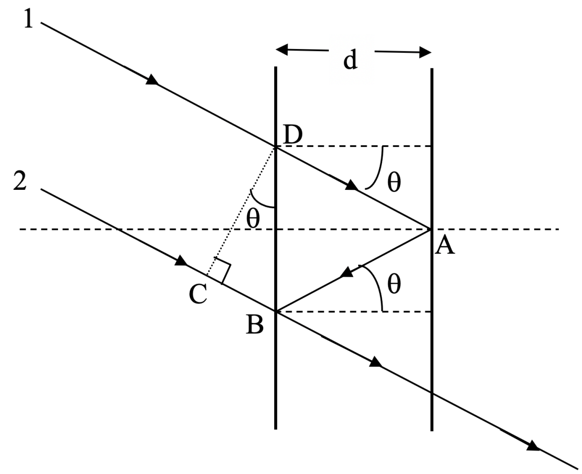


FIG. 4. Reflected and transmitted rays at the parallel surfaces of the elatons. The spacing is d = 1.995 (mm)

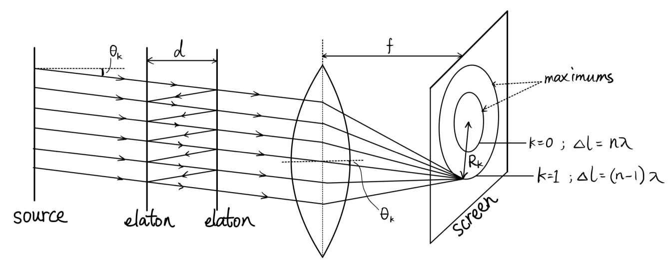


FIG. 5. A sketch of the imaging principle for the Fabry-Perot Interferometer

incoming ray forming an angle θ with the optical axis is partially reflected at point A and meets with the incoming adjacent ray2 at point B. The path difference between the two adjacent rays (Ray 1 and Ray 2) is:

$$\Delta l = DA + AB - CB = 2d\left(1 - \frac{\theta^2}{2}\right) \tag{16}$$

, with the small θ approximation $cos(\theta) = 1 - \theta^2/2 + ...$ For a constructive interference (in fig5):

$$\Delta l = (n-k)\lambda \tag{17}$$

, where n - k is the order of interference with higher values towards the center, n is approximately equal to $\frac{2d}{\lambda}$ for small angle θ[8] and k is the ring number with 0 the smallest ring.

Let θ_k be the angle for k^{th} ring on the screen, the equation 17 becomes:

$$2d\left(1 - \frac{\theta_k^2}{2}\right) = (n-k)\lambda \tag{18}$$

A small change in wavelength λ is magnified as a large change in θ_k because the large n is multiplying the λ.

Depending on the focal length of the convex lens (f), a change in angle corresponds to a change in the radius of the ring on the screen. With small θ_k approximation $\theta_k = \frac{R_k}{f}$, equation 18 becomes:

$$2d\left(1 - \frac{R_k^2}{2f^2}\right) = (n-k)\lambda \tag{19}$$

The value of focal length f can be determined by solving the equations19 for two different rings with their measured radius. Let's choose k = 0 as one of the two rings:

$$2d\left(1 - \frac{R_0^2}{2f^2}\right) = n\lambda \tag{20}$$

, where R_0 is the radius of the innermost ring that we can measure Using equation20 and equation19 for any other ring with k, we can solve for f:

$$\frac{d}{f^2}(R_k^2 - R_0^2) = k\lambda \tag{21}$$

Define a constant C_0:

$$C_0 = \frac{d}{f^2\lambda} = \frac{k}{R_k^2 - R_0^2} \tag{22}$$

With the presence of an external magnetic field, we will see that each original ring will be splitting into 9 different rings according to the theory in Fig3. Each of the three-line groups (π-lines, σ_--lines, and σ_+-lines) has middle rings (transition line 5, 2, and 8 in tableI). For better measurement, we will choose transition line 5 which has a wavelength equal to the original wavelength of the mercury lamp ($\lambda_5 = 546.1nm$). The outer rings (transition line 1, 4, and 7 in tableI) has larger energy hence slightly smaller wavelength (λ_-) and the inner ring (transition line 3, 6, and 8 in tableI) has slightly larger wavelength (λ_+). Using the equation19 for the two splits and subtracting the two equations we have:

$$\frac{d}{R_{k+}^2 - R_{k-}^2} = (n-k)(\lambda_- - \lambda_+) \tag{23}$$

, where R_{k-} and R_{k+} are the radius of the inner and outer rings of each line group. Using the equation15 and the approximation $(n-k)\lambda = n\lambda = 2d$:

$$\mu_B = \frac{hc}{2Bf^2\lambda}(R_{k+}^2 - R_{k-}^2) \tag{24}$$

, where f can be determined by equation22 by doing the measurement with no external magnetic field with $\lambda = \lambda_5$. With $C_0 = \frac{d}{f^2\lambda_5}$, the Bohr Magneton can be determined by measurment for the π-lines:

$$\mu_B = \frac{C_0 hc}{2dB}(R_{k+}^2 - R_{k-}^2) \tag{25}$$

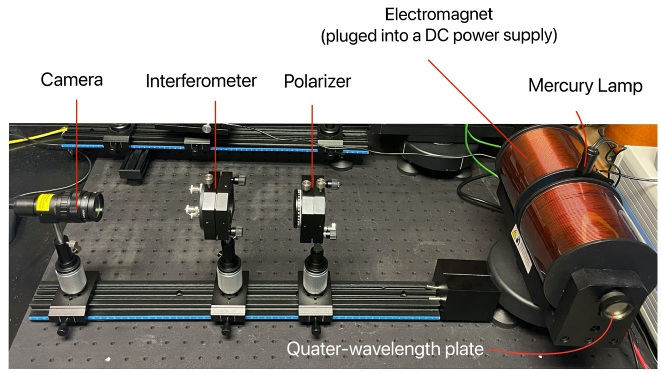


FIG. 6. The Experiment Setup

III. EXPERIMENTAL METHODS

In our experiment, we immersed a mercury lamp into the electromagnet. The electromagnet can be rotated by 90° and in this case, the light from the lamp is traveling through the quarter-wavelength plate. The electromagnet was connected to the Direct Current (DC) supply. The current can be adjusted from 0A to 5A. Between the Fabry-Perot Interferometer and the lamp, we put a polarizer to select the polarization states of the spectral lines that appear on the image from the camera. Fig6 shows all the main materials we used in the experiment.

For run 1, we recorded the changes of the spectral line when the current was increased from 0A to 5A with polarizer at 0°, which is perpendicular to the magnetic field. We observed two groups of triplets for each k value, which was within expectation. According to the theory, these two groups of triplets are the σ-lines. The image is shown in Fig7

For run 2, we recorded the changes of spectral line with polarizer at 90°, which is parallel to the magnetic field. We observed one triplet for each k value. These triplets are the π-lines according to the theory. (see Fig8)

For run 3, we rotated the electromagnet by 90° and recorded the changes of spectral line with a quarter-wavelength plate and the polarizer at 90°, which is perpendicular to the magnetic field. We observed a triplet with a decreased radius.(see Fig9). By comparing these triplets to the two triplets in Run1, we see that these triplets are the σ_--lines. For run 4, we rotated the electromagnet by 90° and recorded the changes of spectral line with a quarter-wavelength plate and the polarizer at 0°, which is also perpendicular to the magnetic field. We observed a triplet with an increased radius.(see Fig9). These triplets are the σ_+-lines by comparing it with Run1. This two runs showed that two lines (σ_- and σ_+) are circularly polarized.

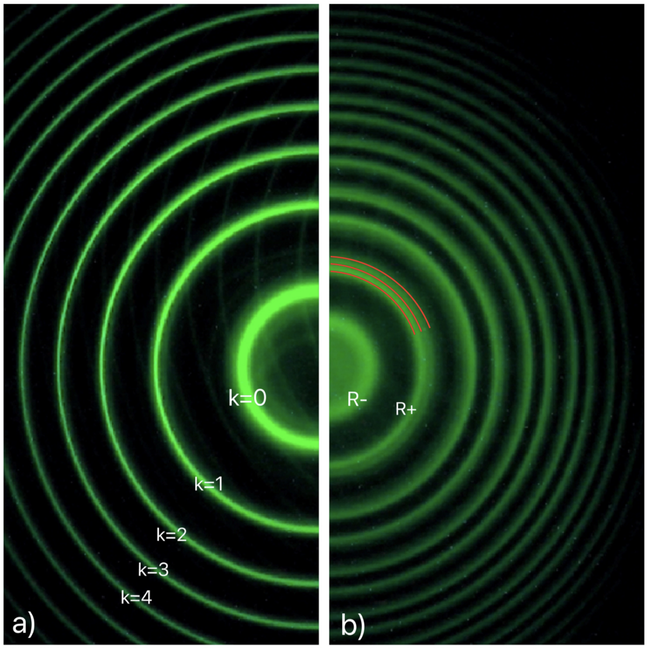


FIG. 7. Run#1 polarizer at 0° perpendicular to the field a)The spectral lines when the current was 0A, no magnetic field. b)The spectral lines when the current was 5A

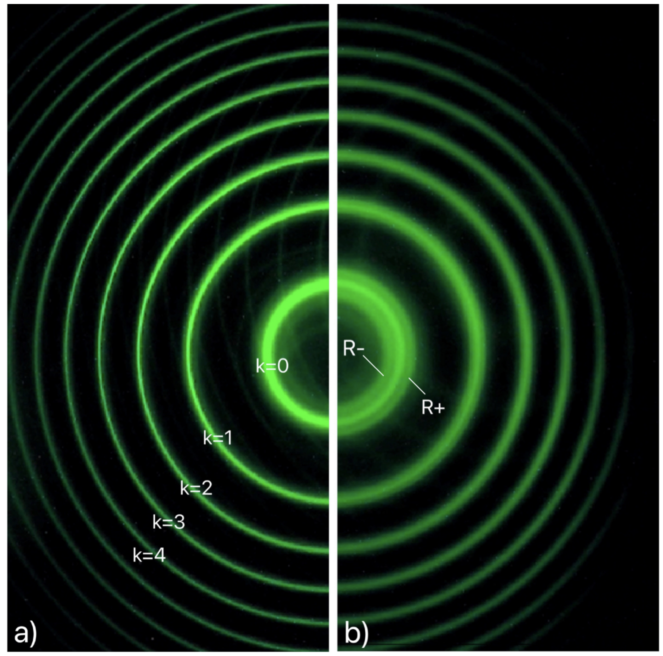


FIG. 8. Run#2 polarizer at 90° parallel to the field a)The spectral lines when the current was 0A, no magnetic field. b)The spectral lines when the current was 5A

TABLE II. Measurement for Run#2

k	$R(m)$	$R_-(m)$	$R_+(m)$
0	0.215	0.178	0.248
1	0.449	0.429	0.466
2	0.598	0.581	0.611
3	0.717	0.707	0.732

IV. DATA ANALYSIS

Before we conducting the measurement for the runs, we immersed a magnetic field detector into the same position as the mercury lamp and recorded the intensity of the magnetic field. We confirmed that the magnetic field is directly proportional to the current, as shown in Fig11. As the current went up, we observed the radius of the splitting for each run increased, which verify that the Zeeman splitting is, indeed, directly proportional to the magnetic field.

We conduct the measurement for Run#2 in Fig8 (π-lines) because this group of lines has the largest intensity and is better for the measurement. The tableII shows the data for the measurement for Run2. Using the equation22 and equation25, we can calculate the Bohr Magneton. The calculation results are shown in the tableIII.

We also conduct measurement for Run#1, where the lines are horizontally polarized (σ-lines). We measured the inner three sets of six rings for the three k order. For each of the set of six rings, we measured the radius of the two rings that have the most intensity, which are the rings that are closest to the original ring. These two rings correspond to the transition 1 and 7 in tableI. The measured data are shown in the tableIV.

TABLE III. Calculateion Results for Run#2 C_0 and μ_B

k	$R(m)$	$R_-(m)$	$R_+(m)$	$C_0(1/m^2)$	μ_B (J/T)
0	0.215	0.178	0.248	NA	9.28×10^{-24}
1	0.449	0.429	0.466	6.436	10.36×10^{-24}
2	0.598	0.581	0.611	6.423	11.19×10^{-24}
3	0.717	0.707	0.732	6.412	11.23×10^{-24}
Average	NA	NA	NA	6.424	10.52×10^{-24}

V. CONCLUSION

In this experiment, we observed and measured the anomalous Zeeman Effect from a mercury lamp. The splitting patterns we observed are agreed well with the theory. The average Bohr Magneton value is close to the theory (9.27×10^{-24}) with an error of +13.4%. Although the average Bohr Magneton value does not quite agree with the theoretical value, the Bohr Magneton value for k = 0 (9.28×10^{-24}) is impressively close to the theory

TABLE IV. Measurement for Run#1

k	$R(m)$	$R_-(m)$	$R_+(m)$
0	0.215	0.146	0.272
1	0.449	0.416	0.479
2	0.598	0.575	0.620

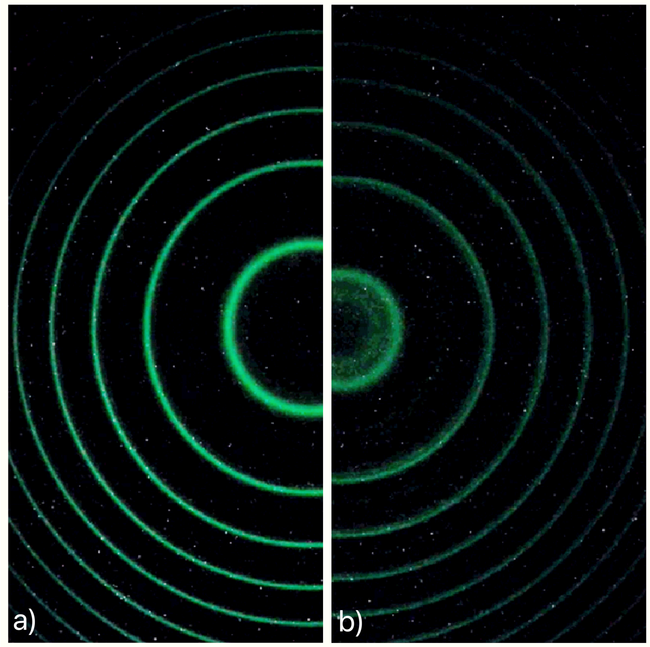


FIG. 9. Run#3 Magnetic field was rotated by 90° with polarizer at 90° perpendicular to the field a)The spectral lines when the current was 0A, no magnetic field. b)The spectral lines when the current was 5A

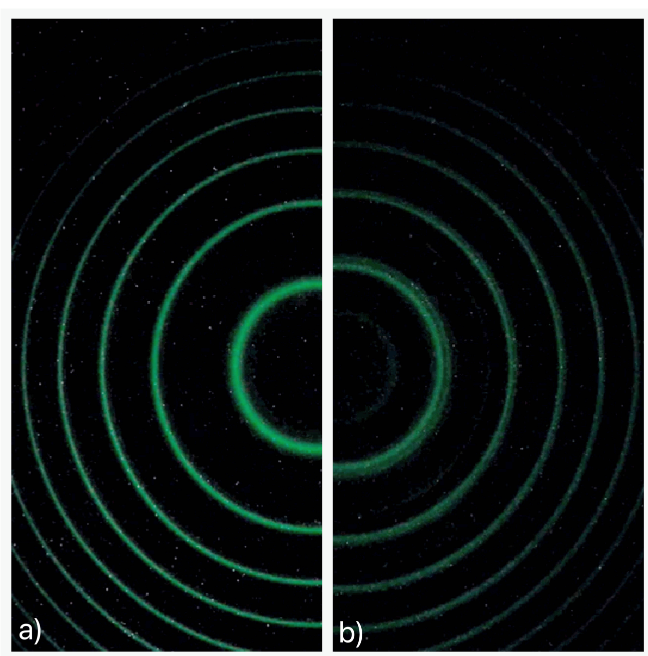
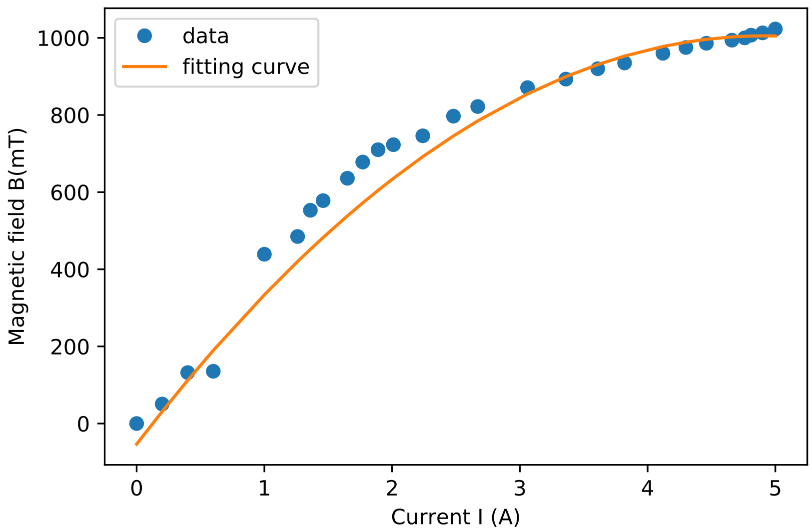


FIG. 10. Run#4 Magnetic field was rotated by 90° with polarizer at 0° perpendicular to the field a)The spectral lines when the current was 0A, no magnetic field. b)The spectral lines when the current was 5A

in an error of only +0.12%. The Bohr Magneton values for k = 1, 2, and 3 are all higher than the theory. This is probably due to the low image resolution for higher k rings. The higher the k, the more uncertainty for the measurement, hence the more error for the calculation results. Also, during the measurement for fuzzy rings, We tend to measure the edge, which will result in a higher radius difference between the $_{-}$ and $_{+}$ rings than the actual value, and lead to higher Bohr Magneton values. To get better results, more precise calibration for the interferometer should be done to get a better resolution.



	Current I(A)	Magnetic field B(mT)
0	0.23	50.6
1	0.44	132.1
2	0.65	135.5
3	1.05	439
4	1.26	485
5	1.36	553
6	1.46	578
7	1.65	636
8	1.77	678
9	1.89	710
10	2.01	723
11	2.24	746
12	2.48	797
13	2.67	822
14	3.06	871
15	3.36	893
16	3.61	920
17	3.82	935
18	4.12	960
19	4.31	975
20	4.46	986
21	4.66	994
22	4.76	1000
23	4.81	1007
24	4.91	1013
25	5.01	1023

FIG. 11. Relationship between Intensity of Magnetic Induction and Current

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